Louay M. Al-Alousi,[1] Ph.D.; Robert A. Anderson,[1] Ph.D.; Diana M. Worster,[2] B.A.; and David D. Land,[3] Ph.D.

Multiple-Probe Thermography for Estimating the Postmortem Interval: II. Practical Versions of the Triple-Exponential Formulae (TEF) for Estimating the Time of Death in the Field

REFERENCE: Al-Lousi LM, Anderson RA, Worster DM, Land DD. Multiple-probe thermography for estimating the postmortem interval: II. Practical versions of the triple-exponential formulae (TEF) for estimating the time of death in the field. J Forensic Sci 2001;46(2):323–327.

ABSTRACT: A simple, reliable, and relatively accurate method for estimating the time since death is described. It is based on the Triple-Exponential Formulae (TEF), which are devised for the first time in this study. The postmortem cooling rate of the brain, liver, and rectum in 117 forensic cases were investigated (Part I). The method can be used in the field as a computer program, reference graph, or reference chart-ruler. The program is simple and can easily be run by any user. There are six reference graphs representing the average brain, liver, and rectal cooling curves for naked and covered body groups. The ruler is designed for the rectal cooling curves for covered and naked bodies. This method requires one temperature measurement of the chosen body site and the environment. The postmortem interval is estimated as a probable value ± a possible range of time estimates with a built-in confidence limit.

KEYWORDS: forensic science, forensic pathology, forensic medicine, time of death, postmortem interval, triple-exponential formulae (TEF), microwave thermography

The problem of estimating the postmortem interval is regularly encountered in forensic practice and the answer to it frequently becomes of decisive value as to the outcome of a suspicious case (1–3,8,12,15).

It is now widely acknowledged among forensic practitioners that the temperature method for estimating the time of death is more reliable, easier to implement, and more likely to be of value than other methods, especially during the first 24 h after death (2,7,9,10,14–16). Nevertheless, this does not mean that methods based on the postmortem rate of cooling are absolutely reliable and accurate as it has been shown that even the best of the cooling models may give significant errors in the estimation of the time since death when it is applied in the field (4–6,10,11,13).

There is, therefore, a continuing need for more accurate and reliable (albeit more complicated) methods for the estimation of the postmortem interval. An equally important objective is to establish statistically the extent of errors inherent in the method. These goals have become more possible and practicable in recent years by the application to forensic medicine of modern techniques for the acquisition, processing, and analysis of data (1–3).

It should be noted that most available temperature-based methods, so far, are difficult to apply in the field, as they require the knowledge of several factors or the measurements of body parameters, which are usually not available for the forensic practitioner to start with. On the other hand, there are methods which are possibly easy to use but are basically flawed.

Experimental

Most methodological aspects of the data collection and analysis have been described earlier (Part I). Temperature difference ratios (Eq 1, Part I) at selected successive intervals after death, for naked and covered body groups, were averaged by simple numerical technique. Values of average and standard deviation were plotted versus postmortem time and areas of the scatters around the averages were shaded. Thus, six reference graphs were obtained.

Using Eq 1 (Part I) and the rectal temperature at the moment of death (T_{b0}) for the naked body group (Table 1), temperature difference ratios were calculated for selected values of rectal and environmental temperatures within a temperature scale ranging from 1 to 36°C. Thus, 36 curves were obtained, which represent the relation between the rectal and the environmental temperatures over a temperature range of 1 to 36°C. Then these curves were plotted to match the average rectal cooling curves for the covered and naked body groups. Thus, the Reference-Chart-Ruler was obtained.[4]

Average parameters (P_1 to P_6, Eq 2, Part I) were implemented in a specially written simple Window-based computer program. To enable this program to calculate the possible errors in the time estimates, the following procedure was performed: at selected postmortem intervals and for each body site, values of the standard deviations were added to and subtracted from the values of the mean temperature difference ratios R. Values resulting from the addition and subtraction were denoted as (U) and (L) respectively (U for upper limit and L for lower limit). U and L values were then fitted

[1] Senior lecturer and consultant forensic pathologist, Senior lecturer (Forensic Toxicology), respectively, Department of Forensic Medicine & Science, University of Glasgow, Glasgow G12 8QQ, UK.

[2] Senior computer advisor, Computing Service Department, University of Glasgow, Glasgow G12 8QQ, UK.

[3] Senior lecturer, Department of Natural Philosophy and Astronomy, University of Glasgow, Glasgow G12 8QQ, UK.

Received 23 Nov. 2000; and in revised form 24 May 2000; accepted 29 May 2000.

[4] A similar Chart-Ruler was presented to the annual meeting of the British Association in Forensic Medicine, London, 1991.

TABLE 1—*Average values for upper limit* (U), *mean* (M), *and lower limit* (L) *parameters of the Triple-Exponential Formulae (TEF) incorporated in the computer program devised in this study for the estimation of the time of death by postmortem measurements of brain, liver, and rectal temperatures.* T_{b0} = *Site-specific body temperature at the moment of death experimentally found in this study and integrated in the program.*

Parameters	Brain*	Liver Covered	Liver Naked	Rectum Covered	Rectum Naked
$M_1^\dagger$	−5.9	−5.86	−6	−5.99	−6
$M_2^\dagger$	−0.10134	−0.14237	−0.1039	−0.19152	−0.18765
$M_3^\dagger$	4.5	5.1	5.1	5.1	5.1
$M_4^\dagger$	−0.119334	−0.14995	−0.08953	−0.20642	−0.14585
$M_5^\dagger$	2.4	1.76	1.9	1.89	1.9
$M_6^\dagger$	−0.08309	−0.05363	−0.1334	−0.084774	−0.22793
U_1	−5.904	−5.9	−6	−6	−5.4
U_2	−0.0364	−0.16698	−0.03044	−0.16195	−0.13651
U_3	4.5048	5.3	5	5.4	6.1
U_4	−0.05618	−0.17356	−0.03925	−0.1656	−0.12052
U_5	2.3992	1.6	2	1.6	0.29958
U_6	−0.0197	−0.04113	−0.02044	−0.06362	−0.07576
L_1	−17.6341	−5.98	−11.7	−13.48	−13.52
L_2	−0.49342	−0.26282	−0.19874	−0.18611	−0.2734
L_3	11.7687	5.53	7.9	5.41	9.2919
L_4	−0.40857	−0.2794	−0.16495	−0.23512	−0.22209
L_5	6.86538	1.45	4.8	9.07	5.2282
L_6	−0.5998	−0.06562	−0.24169	−0.14849	−0.33819
T_{b0}°C‡	26.64	32.7	27.52	36.62§	36.62

* Brain cooling is not affected by covering of the torso.
† M_1-M_6 = P_1-P_6 [Part II].
‡ Site-specific temperature at moment of death [Part I].
§ T_{b0} for naked body group is used instead of that for the covered body group because the former is more representative than the latter.

to triple-exponential equations using the BMDP P3R Program. The formulae thus obtained were also implemented in the program (i.e., our program). Thus, three formulae were used for each site and each group of related cases, the first was the average cooling formula, the second was the upper limit formula, and the third was the lower limit formula as follows:

$$R_M = M_1 e^{M_2 tm} + M_3 e^{M_4 tm} + M_5 e^{M_6 tm} \quad (1)$$

Where M_1 to M_6 (Table 1) are the mean values of the parameters $P_1 - P_6$ (Eq 2, Part I), R_M is the mean value of the temperature difference ratio R (Eq 1, Part I), and tm is the most probable postmortem interval (PMI) estimate.

$$R_U = U_1\, e^{U_2 tu} + U_3\, e^{U_4 tu} + U_5\, e^{U_6 tu} \quad (2)$$

Where U_1 to U_6 (Table 1) are upper limit values of the parameters $P_1 - P_6$ (Eq 2, Part I), R_U is the upper limit value of the temperature difference ratio R (Eq 1, Part I), and tu is the upper limit postmortem interval (PMI) estimate.

$$R_L = L_1\, e^{L_2 tL} + L_3\, e^{L_4 tL} + L_5\, e^{L_6 tL} \quad (3)$$

where L_1 to L_6 (Table 1) are lower limit values of the parameters $P_1 - P_6$ (Eq 2, Part I), R_L is the lower limit value of the temperature difference ratio R (Eq 1, Part I), and tL is the lower limit postmortem interval (PMI) estimate.

The most probable estimated postmortem interval was calculated by the average formula and the range of time estimates was calculated by upper and lower limit formulae. Also implemented in the program were Eq 1 (Part I) and the temperatures at the moment of death (T_{b0}) of the three body sites studied. These were derived empirically by a procedure outlined earlier (Part I). This enables the program to calculate the temperature difference ratio R. Therefore, the user should measure and input temperatures of the chosen body site and the environment as encountered in the field.

Results

Average Curves for Practical Use: Reference Graphs and Chart-Ruler

Through the curve fitting procedures (Part I) and subsequent averaging processes (2,3) the average cooling curves were obtained. These incorporated the scatter of temperature difference ratio (R) values to ± 1 standard deviation. By joining the upper and lower limits of R scatters together and shading the area thus obtained, the errors or ranges of time estimates, which would probably be encountered in the field, can easily be calculated at any point in the graph (Fig. 1). These reference graphs simplified the application of the Triple-Exponential Formulae (TEF) to actual cases in the field. However, there is still a need for the forensic practitioner to calculate the values of R in the field using Eq 1 (Part I). This was considered a drawback and therefore a further simplification was made by introducing the reference chart-ruler (Fig. 2), which can be obtained by a special arrangement with the authors.

A Computer Program for Practical Use

The triple-exponential equation used in this study can easily be solved by a microcomputer or even by a small pocket computer. The average values of the mean, upper limit, and the lower limit of the TEF parameters, which are integrated in the program, are shown in Table 1. The site-specific temperatures at the moment of death (T_{b0}), which are incorporated in the program (Table 1) were described previously (Part I). This enables the program to calculate the temperature difference ratio R. Therefore the user should measure and input temperatures of the body site and the environment as encountered in the field. This program can be obtained by a special arrangement with the authors.

As the scatter of the values of the temperature difference ratio (R) was relatively wide, only one standard deviation was used.

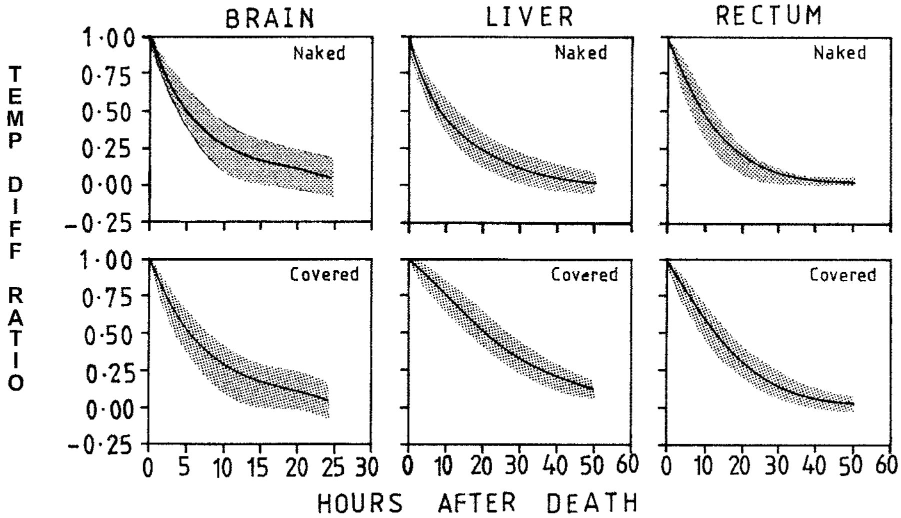

FIG. 1—*Reference graphs for estimating the time of death based on the Triple-Exponential Formulae (TEF) for three body sites in naked and covered body groups. The shaded areas indicate the range of errors corresponding to ± 1 standard deviation.*

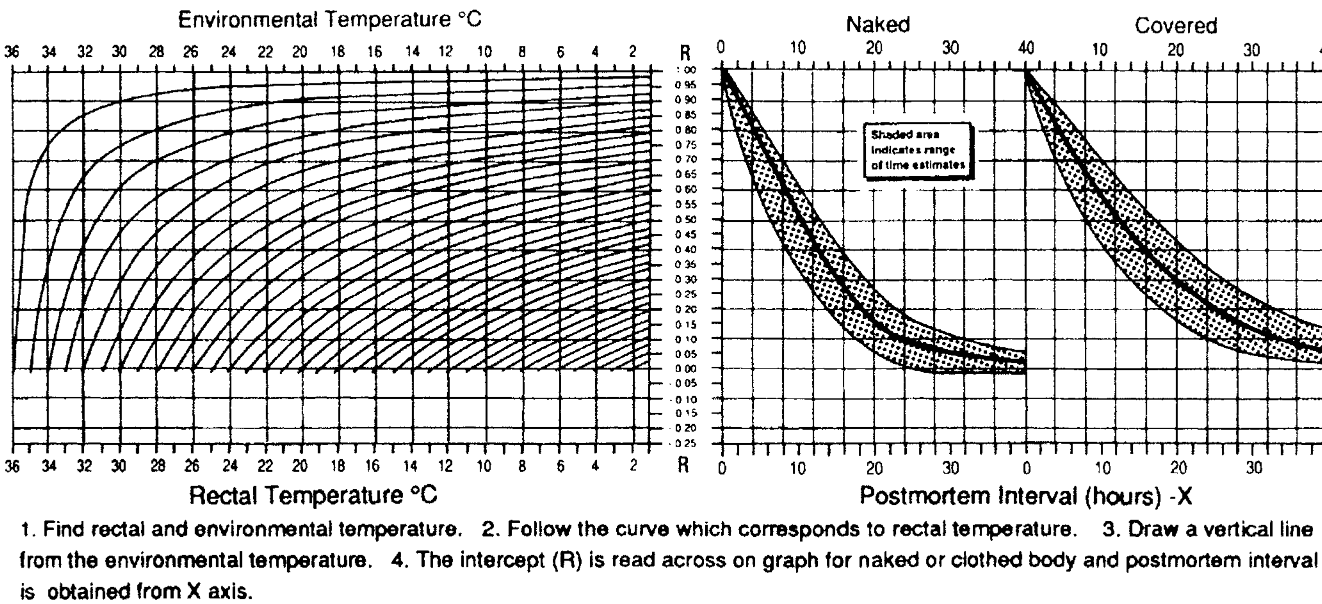

FIG. 2—*Reference Chart-Ruler based on the Triple-Exponential Formulae (TEF) for estimating the time of death for the rectal cooling in the naked and covered body groups.*

This means that the confidence limit of the postmortem interval (PMI) estimations obtained by this method is 68%. In other words, this method is expected to accurately predict the postmortem interval in about two-thirds of the population. However, when the program devised in this study is used, the user can input temperatures of more than one body site. These should be measured at the same time so that the time after death would be estimated according to different average formulae. By averaging the errors in time estimates resulting from these formulae, the precision and reliability of time prediction is greatly improved (Table 2). Thus, two or even three standard deviations can be used and, therefore, the applicability of this method to normal population is widened.

Discussion

General Appraisal of TEF Method

It should be noted that temperature-based methods are not equal in their applicability to the field. Simple models such as the "Rule of Thumb" and "Newtonian Cooling" are perhaps easy to apply in the practice, but have been shown to give enormous errors in time estimates (16). Moreover, from a theoretical point of view, the application of these models to the postmortem cooling is shown to be invalid (2,3). Other methods such as the double-exponential formula and its modifications are based on sound mathematical expression of the actual postmortem cooling curve but they are difficult to be used in practice. For instance, the double-exponential

TABLE 2—*Mean values of the deviation of estimated time from the actual time using the average formulae for the three body sites separately and together in covered and naked body groups at selected intervals after death.*

	Mean Deviation (Hours)							
	Covered Bodies (N = 43)				Naked Bodies (N = 74)			
Actual PM Time (Hours)	Brain	Liver	Rectum	All Sites	Brain	Liver	Rectum	All Sites
1	0.3	0.1	0.5	0.3	0.3	0.1	0.1	0.2
2	0.2	−0.4	0.5	0.1	0.2	0.2	0.4	0.3
4	−0.1	0.1	0.3	0.1	−0.1	−0.2	0.4	0.03
8	0.0	−0.1	0.1	0.0	0.0	−0.4	0.3	0.03
12	0.6	−0.4	0.4	0.2	0.6	−0.5	0.5	0.2
18	4.0	−0.1	1.4	1.8	4.0	−0.2	0.4	1.4
24	6	0.5	1	2.5	6.0	3.7	-0.3	3.1

formula of Marshall and Hoare requires several temperature measurements over a period of many hours and in most occasions for at least 12 h. Also, it requires the measurements of some body parameters such as the weight and the surface area. In addition to that, it needs many constants to be derived for each case. Although some of the modifications and amendments of the double-exponential formula are relatively easier to use, they all inherit almost the same amount of errors in time estimates as well as the requirements for the measurements of the body weight and height, which are not readily available for the forensic practitioner in the field (17,18). Moreover, most of these methods have not been evaluated statistically on a significant number of cases and, therefore, the confidence limits of their use are not known.

The complexity of the problem of estimating the time of death is now apparent as many factors affect the postmortem cooling and sometimes interact in an unpredictable and irregular manner. This has caused some authorities in this field to suggest rightly that a good method for the timing of death should be able not only to provide a point of probable time estimate, but a bracket or window of possible estimates, to enable the law enforcement agencies to be flexible in dealing with such a complicated issue (16). Unfortunately, most of the available temperature methods, so far, are unable to do so.

The current study represents a novel approach to the use of cooling models, in that:

1. It used microwave thermography to measure the temperatures of internal body organs by noninvasive techniques as described previously (1).
2. It also used computer-based methods extensively for data acquisition and processing (Part I).
3. Useful practical versions of the Triple-Exponential Formula (TEF) were developed and evaluated for the first time. These versions are very easy to use in the field. They require one temperature measurement of the body and the environment. Also, they give results which are relatively more accurate and reliable than the existing methods (2,3).
4. The error in time estimates resulting from the use of the method was assessed statistically on a relatively large number of cases (117 forensic cases).
5. The temperatures of each body site at the moment of death were studied and calculated by extrapolation from empirical data (Part I).
6. The shape of the cooling curve in three body sites was extensively evaluated both mathematically and statistically and the concept of the temperature plateau was tested (1–3).

The Use of Reference Graphs

The solution of the average formulae developed in this study can be carried out manually using a reference graph, a reference chart-ruler or, more accurately, using a micro or even a pocket computer. It is, therefore, entirely possible to use this method in the field if the portable version of the microwave system, which has recently become available, is used. For this purpose reference graphs were prepared for each body site in the naked and the covered body groups (Fig. 1). Each graph contains the average cooling curve for a body site in either the covered or the naked group. In addition to that, each graph contains a shaded area around the average curve. This area represents values of the standard deviation and indicates the possible errors in time estimates or the range of time estimates, which are likely to be encountered in practice. It is important to note that because the scatter of the values of the temperature difference ratio is originally wide, only one standard deviation is used and therefore these errors can, from a statistical point of view, be encountered in 68% of the cases in practice. In other words, the confidence limit of estimating the postmortem interval according to these graphs is 68%.

For the reference graph to be used, the temperature of a body site and the temperature of the environment are first measured in degrees centigrade. The second step is to calculate the temperature difference ratio R from Eq 1 (Part I). The temperature of the chosen body site at the moment of death (T_{b0}) should be obtained from previous publications (Part I). The resulting average time after death and the probable range of error in the time estimate can then easily be measured.

The Use of Reference Chart-Ruler

The necessity to work out the value of temperature difference ratio (R) by the forensic investigator is considered as a drawback, however small, of the reference graphs' technique. Therefore the chart-ruler was designed to avoid any calculation. To use the chart-ruler, follow the instructions printed on the ruler itself. The shaded area gives the possible range of time estimates. For example, if the rectal temperature is 23°C and the air temperature is 19°C, follow the curve which corresponds to rectal temperature 23°C, then draw a vertical line from environmental temperature 19°C to the rectal temperature curve; the intercept, in this case, would correspond to R value at 0.25. Therefore, the estimated postmortem interval in a naked body is probably 18 h, ranging from 15.5 to 23 h after death and, similarly, in a covered body the estimate is probably 23 h, ranging from 18 to 28 h after death. The confidence limit is still the same 68%. The ruler was designed for the rectal cooling only because this, so far, is the site of choice for the measurements of postmortem temperatures, taking in consideration that the microwave thermograph is not available for general use yet.

The Use of Computer Program

To enable practicing pathologists or forensic examiners to use the triple-exponential formulae developed in this study for more accurate estimation of the time since death, a specially written simple computer program was prepared. The program operates in any Window operating system. Following the instructions given by the program, the investigator is asked to input the temperature of a body site and the temperature of the environment and to indicate which body site is used (rectum, brain, or liver). Other data required for the prediction of the postmortem interval (Table 1) are built in the program itself. It should be observed that whenever the temperature of the environment is required this should be measured at about 2 m distance from the body to avoid artifacts due to the heat of the body. It is worth noting that when the temperature of a body site approaches that of the environment, the possible errors or the range of time estimates become wide and therefore, the results are less reliable. One-standard deviation values were incorporated in the program and therefore the confidence limit of the time estimates obtained from the program is 68%. If the Microwave Thermography System (1), or a similar device, is available the program would be able to accept inputs of the brain, liver, and the rectal temperatures together and, thus, a more accurate and reliable estimate of postmortem interval can be obtained (Table 2). At such conditions two, or even three, standard deviations can be used and, therefore, the accuracy and reliability of this method is strengthened and its applicability to the field is widened.

Finally, this study has emphasized the view that the problem of time of death is very complex and, therefore, further studies are required to scrutinize the influence of various body and environmental factors on postmortem cooling. We believe that the method devised in our study provides significant improvements as to the accuracy and reliability of time estimates over the existing methods. However, it is important to note that this method aims primarily at providing the law enforcement agencies with an investigational tool and, therefore, when the method is employed for evidential purposes it should be used with caution. We think that this method can be applied to a wide range of forensic cases and it would be useful for most of the situations encountered in usual forensic practice. However, it should be applied with care to bodies discovered under extreme weather conditions.

References

1. Al-Alousi LM, Anderson RA, Land DV. A non-invasive method for post-mortem temperature measurements using a microwave probe. Forensic Sci Internat 1994;64:35–46.
2. Al-Alousi LM, Anderson RA. Microwave thermography in forensic medicine. Police Surgeon 1986;30:30–42.
3. Al-Alousi LM, Anderson RA. Post-mortem interval by microwave thermography. In: Caddy B, editor. Users of the forensic sciences. Edinburgh: Scottish Academic Press, 1987;101–15.
4. Brown A, Hicks B, Knight B, Nokes LDM. Determination of time since death using the double exponential cooling model. Med Sci Law 1985;25(3):223–7.
5. Green MA, Wright JC. Post-mortem interval estimation from body temperature data only. Forensic Sci Internat 1985;28:35–46.
6. Lyle HP, Cleveland FP. Determination of the time of death by body heat loss. J Forensic Sci 1956;4:11–23.
7. Mant AK. Recent work on changes after death: lapse of time from chemical changes in cerebrospinal fluid. Factors influencing changes after burial, fluidity of blood after death. In: Simpson K, editor. Modern trends in forensic medicine. London: Butterworths, 1953;80–98.
8. Mant AK, editor. Taylor's principles and practice of medical jurisprudence. 13th ed. Edinburgh: Churchill Livingstone, 1984;128–55.
9. Marshall TK. Changes after death. In: Camps FE, Robinson AE, Lucas BGB, editors. Gradwohl's legal medicine. 3rd ed. Bristol: John Wright & Sons Ltd, 1976;718–1000.
10. Mason JK, editor. Forensic medicine for lawyers. 3rd ed. London: Butterworths, 1995;56–66.
11. Nokes LDM, Brown A, Knight B. A self-contained method for determining time since death from temperature measurements. Med Sci Law 1983;23(3):166–9.
12. Oever RVD. A review of the literature as to the present possibilities and limitations in estimating the time of death. Med Sci Law 1976;16(4):269–76.
13. Paton D. Time of death (letter on the unreliability of the "usual calculation" and the cooling curves according to Gradwohl's). Police Surgeon 1970;15:116.
14. Polson CJ. The signs of death. In: Polson CJ, Gee DJ, Knight B, editors. The essentials of forensic medicine. 4th ed. Oxford: Pergamon Press, 1985;3–39.
15. Simonson J, Voigt J, Jeppeson N. Determination of the time of death by continuous post-mortem temperature measurement. Med Sci Law 1977;17(2):112–22.
16. Knight B. Forensic pathology. 2nd ed. London. Sydney. Auckland: Arnold, 1996;51–95.
17. Hiraiwa K, Ohno Y, Kuroda F, Sibetan IM, Oshida S. Estimation of postmortem interval from rectal temperature by use of computer. Med Sci Law 1980;20(2):115–25.
18. Henssge C. Death time estimation in case work: 1 The rectal temperature time of death nomogram. Forensic Sci Internat 1988;38:209–36.

Additional information and reprint requests:
Dr. Louay M. Al-Alousi, Ph.D.
Department of Forensic Medicine & Science
University of Glasgow
Glasgow G12 8QQ, UK